**Course Description**

This course explores the far-reaching applications in science of a simple but remarkably insightful equation. First introduced in 1764, Bayes’ formula elegantly embodies the scientific method, in which predictions made from hypotheses are put to the test, and inferences regarding the relative merits of the hypotheses are drawn from the resulting data. Bayesian inference is increasingly used as a powerful research tool by scientists in all disciplines. It provides a sophisticated method for drawing inferences from data, used both for statistical analysis and as a model of human brain function. Students will apply Bayes’ formula to gain insight into scientific and everyday reasoning. They will learn modern statistical analysis techniques, derived from Bayes’ formula, that provide a coherent and attractive alternative to the conventional p-value methods. They will be introduced to the Bayesian foundations of signal detection theory and to Bayesian models of sensory perception. This conceptually sophisticated course is presented in an interactive and mathematically accessible fashion.

Psych 730 and Psych 4KK3: Graduate students taking Psych 730 are required to attend all lectures and complete all work along with the undergraduates in Psych 4KK3. The grad students will write more advanced versions of the midterm test and final exam and will complete two additional assignments involving extensive reading and/or data analysis.

**Instructor**

Dr. Daniel Goldreich (goldrd@mcmaster.ca)

**Teaching Assistants**

Keon Allen (allenks2@mcmaster.ca)

Kyle Gauder (gauderkd@mcmaster.ca)

**Meetings**

Lecture: Fridays, 2:30 - 5:20 PM (ETB 238).

Tutorial (optional): Mondays, 1:30 - 2:20 PM (T13 105).

**Office Hours**

Prof's Drop-In Office Hour: Wednesdays, 4:30 - 5:20 PM (PC 413).

TA Office Hours by Appointment: email a TA to arrange a meeting.

**Materials**

Required Textbook: Goldreich, D. (2019) Introduction to Bayesian Inference (available on Avenue at start of semester)

Students are responsible for reading each assigned textbook chapter in its entirety. Some homework and exam questions may be based on material in the book that was not covered during lectures. Note: The book is a work in progress and may be modified slightly throughout the semester. If you are planning on printing the book, please wait until we reach the corresponding course section before printing each chapter, to ensure that you print the most recent version.

Required Calculator: The McMaster Standard Calculator (Casio FX-991) is permitted in exams and for in-class Brain
Boggler exercises unless otherwise specified. This calculator is available at the McMaster Campus Store. No other calculator is permitted.

**Objectives**

Upon completion of this course, the student will understand:

- the concept of conditional probabilities
- probability sum and product rules
- the meaning of each element of Bayes' formula: prior, likelihood, and posterior
- key features of the scientific method that emerge from the application of Bayes' formula.
- how Bayesian inference differs from frequentist statistical methods.
- how to apply Bayesian inference to clinical diagnostic testing and genetics scenarios.
- how to use Bayesian inference to estimate the values of binomial, Poisson, and Gaussian parameters
- how to use Bayesian inference for logistic, Poisson, and linear curve-fitting
- how to do robust Bayesian parameter estimation
- how to perform Bayesian model comparison
- applications of Bayesian inference to the modelling of human sensorineural responses and perception
- the foundations of Bayesian decision theory

**Weekly Topics**

The table below shows weekly lecture topics, textbook readings, and homework assignments. Students are encouraged to read the assigned material prior to the class period. Prior to each class, the instructor will post the majority of the slides to be shown in lecture. HW = homework assignment, GS HW = grad student homework assignment.

<table>
<thead>
<tr>
<th>Week</th>
<th>Topic</th>
<th>Assignment Due</th>
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<tbody>
<tr>
<td></td>
<td><strong>Part 1: Basic Bayes</strong></td>
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<tr>
<td>Jan 10</td>
<td>Basic Bayes: coins, apples, and other interesting items (Ch. 1)</td>
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<tr>
<td>Jan 17</td>
<td>Basic Bayes: medicine and genetics (Ch. 1)</td>
<td>HW 1</td>
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<tr>
<td>Jan 24</td>
<td>Basic Bayes: critical thinking (Ch. 1)</td>
<td>HW 2</td>
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<td>Jan 31</td>
<td>Parameter estimation: binomial, Poisson, and Gaussian (Ch. 2)</td>
<td>HW 3</td>
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<tr>
<td>Feb 7</td>
<td>Curve fitting: logistic, Poisson, and linear regression (Ch. 3)</td>
<td>HW 4</td>
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<tr>
<td>Feb 14</td>
<td>Midterm test (2 hours)</td>
<td>HW 5</td>
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<td><strong>Part 2: Advanced Bayes</strong></td>
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<td>Feb 28</td>
<td>Robust estimation (Ch. 4)</td>
<td>GS HW A</td>
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<td>Mar 6</td>
<td>Advanced methods in parameter estimation (Chs. 5, 6)</td>
<td>HW 6</td>
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<td>Mar 13</td>
<td>Model comparison (Ch. 7)</td>
<td>HW 7</td>
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<td>Mar 20</td>
<td>Bayesian brain I (Ch. 9)</td>
<td>HW 8</td>
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<td>Mar 27</td>
<td>Bayesian brain II (Ch. 10)</td>
<td>HW 9</td>
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<td>Apr 3</td>
<td>Bayesian decision theory (Ch. 11)</td>
<td>HW 10</td>
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<td>Apr 7</td>
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<td>GS HW B</td>
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<tr>
<td>TBA</td>
<td>Final exam (comprehensive, 2.5 hours)</td>
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Evaluation

Students will be graded on in-class critical thinking activities called Bayesian Brain Bogglers, weekly homework assignments, a midterm test, and a comprehensive final exam. The student's course percentage score is a weighted average of the following five items:

<table>
<thead>
<tr>
<th>Item</th>
<th>Weight</th>
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</thead>
<tbody>
<tr>
<td>Bayesian Brain Bogglers</td>
<td>5%</td>
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<tr>
<td>Homework Assignments</td>
<td>25%</td>
</tr>
<tr>
<td>Midterm Test</td>
<td>25%</td>
</tr>
<tr>
<td>Comprehensive Final Exam</td>
<td>45%</td>
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</tbody>
</table>

**Bayesian Brain Bogglers**

The brain bogglers are in-class exercises designed to encourage critical thinking about Bayesian inference. Some of the brain bogglers will require a calculator; the student should bring the McMaster Standard calculator (Casio FX-991) to class. Typically, one brain boggler will be given per week, though some weeks may have more. Unless otherwise announced, each brain boggler exercise is worth 2 points. Each answer will typically receive either zero, half, or full-credit (0, 1, or 2 points). The student's brain boggler percentage score, used in the student's course percentage score calculation, is the number of brain boggler points earned divided by the total number of points possible.

**Homework Assignments**

Homework assignments are linked on the schedule table, and are due at the beginning of class each week. Late homework returns will not be accepted. Students are encouraged to type their answers whenever possible. Students may collaborate as they work on the homework assignments, but they must submit their assignments individually.

The point value of each question will be indicated on the homework assignment. The score for the entire homework assignment will always be reported on a 0-to-100% scale. For example, suppose an assignment has three questions, worth 2, 4, and 4 points each. If a student earns half credit on the first question, full credit on the second, and half credit on the third, then the student's score will be \(1 + 4 + 2 = 7\), and the score will be reported as 70%.

Graded homework assignments will be available for pick-up in class and in the prof's weekly drop-in office hour.

At the end of the term, the student's lowest homework assignment score will be dropped. The average of the student's remaining homework assignment scores will then be calculated. This is the student's course homework score.

Graduate students taking Psych 730 will complete two additional homework assignments (identified as GS HW A and GS HW B in the schedule table). These assignments, which will involve extra reading and/or data analysis, are intended to deepen the graduate students' understanding of the material covered in each of the two course sections. These assignments will be weighted equally with the other assignments in the calculation of the student's course homework score. Grad students must complete both GS HW assignments (A and B). The lowest GS HW grade will not be dropped (only the lowest of the 10 other assignments will be dropped).

**Midterm Test and Final Exam**

The student should bring the McMaster standard calculator (Casio FX-991) to all exams. Only the McMaster standard calculator will be allowed.
Graduate students taking Psych 730 will write somewhat more advanced versions of the midterm test and final exam than the undergraduates taking Psych 4K3.

Each test / exam is given a mark on a scale from 0 - 100%.

Create-a-Question: As an optional exercise, students are encouraged to try to generate an excellent exam question of their own. Please email the instructor with your proposed exam question and answer. Questions that are carefully and articulately worded, and that probe student understanding of important concepts, will be considered for inclusion. The instructor will not inform you in advance of the exam whether your question will be used, and will not provide feedback as to whether your answer is correct. If it is used on an exam, your name will not be attached to your question, but a note will be attached to indicate that the question was student-generated. Furthermore, if it is used, your question may be edited and/or otherwise modified by the instructor.

Extra Credit (Everyday Bayes!)

Opportunities for Bayesian inference occur repeatedly in everyday life, and it is both fun and educational to become aware of these daily events. Students are encouraged to submit their Everyday Bayes examples for extra credit. Each student is allowed a maximum of two Everyday Bayes submissions per semester: one submission prior to the midterm test (deadline: 24 hours before the test start time) and one submission after the midterm test (deadline: last day of classes of the semester). The submission that occurs prior to the midterm test must be based on a concept that was covered in lecture prior to the midterm test (e.g., basic Bayes, parameter estimation, or etc.); the submission that occurs after the midterm test must be based on a concept that was covered in lecture after the midterm test (e.g., robust inference, model comparison, or etc.). In order to generate ideas for high-quality Everyday Bayes submissions, students are encouraged to keep a daily journal of their everyday Bayesian observations.

To earn the extra credit point, an Everyday Bayes submission must report an astute observation regarding the use of Bayesian inference in everyday life, correctly worked out as a numerical example. The example must be typed and emailed to the professor in PDF format. The example must include the use of Bayes’ formula, be clearly and correctly explained, and be correctly calculated and interpreted. In order to earn the extra credit point, the assignment must have no errors or flaws: any error in a description, procedure, calculation, or interpretation will result in failure to earn the extra credit point. No partial marks will be given.

Any extra credit point earned will be applied as an extra percentage point to the student's upcoming test or exam score. Scores earned on exams will not exceed 100%. In the event that a student's midterm exam score would exceed 100% due to extra credit points earned, the score in excess of 100% will be held over to be applied to the student's final exam percentage score (again, up to a maximum score of 100%).

As a learning opportunity for all students in the course, the professor reserves the right to post any Everyday Bayes submission, whether or not it has earned the extra credit point. The post will include the name of the student who submitted the Everyday Bayes paper, and will generally include a statement by the professor as to whether the student earned the extra credit point or not, and why. The post may include comments placed by the prof within the student's submitted PDF file.

Course Grade Calculation

The following formula is used to calculate the student's course percentage score:

\[
\text{Course percentage score} = (\text{course Brain Boggler percentage score})(0.05) + (\text{course Homework percentage score})(0.25) + (\text{Midterm test percentage score})(0.25) + (\text{Final Exam percentage score})(0.45).
\]

The student's course grade will be determined from the student's course percentage score, as follows: A+ (90-100), A (85-89), A- (80-84), B+ (77-79), B (73-76), B- (70-72), C+ (67-69), C (63-66), C- (60-62), D+ (57-59), D (53-56), D- (50-52), F (0-49).
Policies

Recording

Photographs and video recordings are strictly prohibited. Students may make audio recordings of the lectures, for personal use only and not to be posted online, emailed, distributed or otherwise shared. Students should inform the instructor in advance if they wish to make an audio recording.

Cell Phone Use

Cell phone use, including texting, is prohibited in the classroom. Students who need to use their cell phones should leave the classroom and return when they have finished. Cell phone use in the classroom is distracting to the professor and to nearby students, and studies have shown that students who use their phones in class learn less and obtain lower grades. For more information, see:

Weimer M (2012) Students think they can multitask. Here’s proof they can’t. Faculty Focus.

Laptop Computer Use

Students who use laptop computers are expected to do so for valid classroom purposes only (i.e., taking notes and displaying the posted lecture slides). Studies have shown that students who use laptops to engage in activities that are irrelevant to the course (such as social media or internet browsing) distract both themselves and their fellow students, and suffer from diminished understanding of the course material. For more information, see:

Weimer M (2012) Students think they can multitask. Here’s proof they can’t. Faculty Focus.

Email

In any email you send to the instructor or any teaching team member, please write "Psych 4KK3" or "Psych 730" in the subject line. Your email should concern logistical course issues only (e.g., to request an appointment, to ask for clarification regarding the due date of an assignment, etc.). Please do not email us with Bayesian inference content questions; we will not answer such questions by email. Instead, specific content questions should be asked in the tutorials or in class. In addition, you are strongly encouraged to ask questions on the online discussion forum, where other students can help to answer them.

Assignment Submission

Homework assignments are to be submitted at the front of the classroom before the start of lecture. You are encouraged to type your assignment answers, but legible handwritten answers are also acceptable. Late submissions will not be accepted. If a student is unable to attend class because of illness or transportation problems, but has completed the homework assignment, then the student should submit the homework assignment by email, prior to the start of the class period, to the instructor (goldrd@mcmaster.ca). Late email submissions will not be accepted.

Missed Work

Please see the university policy statement concerning missed work. Students who are absent for no more than three days may report their absence, once per term, without documentation, using the McMaster Student Absence Form (MSAF). Please note that the MSAF may not be used for term work worth 25% or more of the course grade, nor can it be used for the final examination. This means that the MSAF cannot be used for the midterm test or the final exam in this course. Absences of more than three days must be reported to the student's Faculty/Program office, with documentation, and relief from course work may not necessarily be granted. A mark of zero will automatically be entered for all missed work until the instructor receives notification from the MSAF system or the student's Faculty Office, and is contacted by the student to discuss how to remedy the missed work situation. It is the student's
responsibility to learn all material that the student has missed for any reason. This can be done by reading the posted lecture notes and assigned textbook chapters, by consulting with classmates, and by attending office hours.

**Academic Integrity**

As a student, you are expected to behave honestly and ethically at all times. According to McMaster University's Academic Integrity Policy, you are engaging in academic dishonesty if you "knowingly act or fail to act in a way that results or could result in unearned academic credit or advantage" (Academic Integrity Policy, p. 6). This behaviour can result in serious consequences, such as a grade of zero on an assignment, loss of credit with a notation on the transcript that reads “Grade of F assigned for academic dishonesty,” and/or suspension or expulsion from the university. It is your responsibility to understand what constitutes academic dishonesty. The following are just three forms of academic dishonesty:

1. Plagiarism.
2. Improper collaboration.
3. Copying or using unauthorized aids in tests and examinations.

For more information on academic dishonesty and academic integrity, please read the Academic Integrity Policy: http://www.mcmaster.ca/academicintegrity

**Note Regarding Course Dates and Deadlines**

The instructor and university reserve the right to modify elements of the course during the term. The university may change the dates and deadlines for any or all courses in extreme circumstances. If such modification becomes necessary, reasonable notice will be given. It is the responsibility of the students to check their McMaster email and course websites weekly during the term and to note any changes.