Investigation of an Extended Climate-Economy Model

A thesis presented in partial fulfillment of the MATH 4P06 course

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Abstract

We aim to investigate the intricacies of a robust climate-economy, stock-flow consistent model, and the effects of rising temperatures on this system. Building upon the prior research of Bovari et al. (2017) [1], we extend their macroeconomic module by adding inventory dynamics, explicit consumption, and active public and banking sectors. Following similar numerical simulation techniques as those described in [1], we analyze the dynamical behaviour of our model in the case of the United States. We observe that our results do not display the rich behaviour of previous Keen-based literature, instead displaying an economic collapse in the long term. We conclude by identifying future research avenues, and proposing improvements to our current model.
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Plots of important ratios from 2016 to 2400, including the wage share ($\omega$), the employment rate ($\lambda$), the profit share ($\pi_f$), and the private debt ratio ($d$). Simulations employed no damage curve, and a low real carbon tax.

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Acknowledgments

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1 Introduction

The most notable and existential issue of our time is centered around our rapidly changing climate, and the litany of resulting economic effects. As the world grapples with the possibility of a climate-induced economic catastrophe (to say nothing of the devastating individual climate catastrophes), governments and elected officials turn to Integrated Assessments Models (IAMs) to inform their decisions.

Our research is largely motivated by the work of Bovari et al. (2017) [2] and (2018) [1], and their attempts to investigate the relationship between rising temperatures, climate-induced damages, and an over-indebted private sector. Specifically, their models enact policies that aim to prevent exceeding two significant thresholds: (i) a temperature anomaly target of $+2\degree$C, as proposed in the Paris Agreement, and (ii) a private debt-to-output ratio of 2.7, over which the liabilities of the private sector will outweigh the assets, potentially giving rise to a systematic cascade of defaults.

Public policy plays a critical role in these continuous-time, stock-flow consistent models. In Bovari et al. (2018) [1], it is assumed that the public sector can levy carbon taxes on the emissions of firms, in line with the suggested growth path of the Stern-Stiglitz Commission (Stern and Stiglitz, 2017 [3]). Taxing carbon emissions and subsidizing green technology will decrease the amount of emissions by the private sector, thus ultimately mitigating the level of temperature increase, as well as the subsequent damages.

We follow their modelling decisions regarding emissions, carbon pricing, and abatement, and we also follow their climate module, as well. Our research differs from theirs, however, as we introduce inventory dynamics, explicit consumption, and active public and banking sectors into our model. Furthermore, we only test our model for the United States, in order to simplify data collection and to have easily comparable historical trends. Our goal is to create a robust, more realistic macroeconomic module that is capable of further understanding the drivers of economic collapse, and the possible measures of avoiding them.

The paper is organized as follows. Section 2 provides a brief review of relevant literature. Section 3 introduces the macroeconomic and climate modelling framework that serves as the basis for our simulations and analysis. Section 4 presents the results of our model. Section 5 offers analysis and interpretation of our results, and outlines some immediate next steps for research, while Section 6 concludes.

2 Related literature

Our research has its roots in the work of American economist Hyman Minsky, without whom no proper literature review in this field can begin. Traditional general equilibrium or optimal behaviour economic models assume that the economy is inherently stable, and will exhibit sustained growth paths so long as there is not some sort of external shock on the system. These models are ineffective at dealing with the potentially disastrous level of economic and financial damages caused by climate change, and so we look to Minsky’s work for motivation.

In Minsky (1986) [4], he postulated that our financial systems are intrinsically unstable in his now-famous “Instability Hypothesis”. He argued that due to this inherent instability, we should expect that financial crises are an expected outcome of our economic system. His hypothesis was later modelled by economist Steve Keen in his 1995 paper [5]. The Keen model is based off of the Goodwin Lotka-Volterra growth model, introduced in 1967 [6], and attempts to quantify Minsky’s idea of inherent instability. This three-dimensional model analyzes the time-evolution of important economic ratios such as the wage and private debt shares of a closed economy, as well
the employment rate.

This model was later found to contain two locally stable long-run equilibria by Grasselli and Costa Lima (2012) [7]. The “good” equilibrium is characterized by a finite private debt ratio, and non-zero wages and employment, whereas the “bad” equilibrium is characterized by vanishing wages and employment, and an infinite private debt ratio. In the past decade or so, there have been many stock-flow consistent models that build upon the work of Keen, such as Godley and Lavoie’s “Monetary Economics” (2007) [8], and in the field of ecological macroeconomics, the work of Dafermos et al. [9] [10].

Much of the work regarding the modelling of climate change originates from William Nordhaus’ Dynamic Integrated model of Climate and the Economy (DICE) [11]. This was the first climate model to introduce a closed feedback loop, taking emissions, the carbon cycle, radiative forcing, temperature, and damages into account. Furthermore, the DICE model made major strides in the field of climate control, specifically in testing different methods and policies for slowing climate change, along with the costs and benefits of these methods. Uncertainties regarding key parameters such as labour productivity growth, the equilibrium climate sensitivity, and the inertia of the climate system were also introduced, and in this model, we follow his most recent assumptions (Nordhaus, 2016 [12]).

Nordhaus found that even the most stringent abatement and taxation policies would be unable to slow climate change to a point where our economic systems may be spared of any damages. In his 1994 book [11], he observes that the momentum of global emissions of carbon dioxide (CO2) and the inertia of the climate system will be too much for the global economy to overcome, and that climate-generated damages are inevitable.

There has thus been a lot of research performed on the convexity and impact of the damage function. The DICE model employs a quadratic damage curve in [11], and therefore produces damages that are probably unrealistic, as argued by Weitzman (2012) [13] and Dietz and Stern (2015) [14]. The Weitzman and Stern specifications increase in convexity, and are thus more suitable for examining the impact of possible climate catastrophes on our economic and financial systems.

Other notable work regarding damages originates from Burke et al. (2015) [15], in which it is posited that as temperature increases, workers are more prone to illness, disease, and fatigue, thus decreasing their productivity. In our research, however, we will only employ the Nordhaus and Stern specifications, allowing for damages to output in the case of the former, and damages to both output and capital in the case of the latter.

Our research is mostly grounded in the work done by our colleagues at the Agence Francaise de Développement (AFD), in Paris, France. More specifically, we employ many of the same modelling techniques as those presented in Bovari et al. (2017) [2] and Bovari et al. (2018) [1]. In their 2017 paper, they develop a continuous-time, stock flow consistent macroeconomic model that considers the economic impacts of climate change, and the potentially catastrophic idea of over-indebtedness in the private sector. They find that climate change will not only cause damages by reducing output and capital, but by also forcing the private sector to leverage in order to compensate for their losses. This leads to a potentially explosive private debt, which then produces a major economic collapse.

Their 2018 work [1] builds upon their research from earlier, by further examining the trade-off between mitigating climate damages and avoiding an explosive private debt. Part of the extension of their earlier work includes performing a sensitivity analysis on four of their key economic and climate variables, similar to the work done in Nordhaus (2016) [12]. They find that as the level of public intervention increases, so does the probability of achieving the +2°C target outlined in the Paris Agreement, and that even though private debt does indeed increase, the risk of over-indebtedness is fortunately limited.
We then attempt to build on their work by introducing inventory and consumption dynamics, inspired by the work of Grasselli and Nguyen Hun (2018) [16]. Their research follows many of the same core assumptions of Grasselli and Costa Lima (2012) [7] and Bovari et al. (2017) [2], for example, but differs by assuming a disequilibrium exists in the goods market. This disequilibrium between expected sales and private demand is accounted for through the use of inventories, specifically unplanned changes in inventory.

Furthermore, our work establishes a more robust banking sector, along with an active central banking sector, as well. We follow some of the ideas presented in Grasselli and Lipton (2018) [17], as well as much of the framework outlined in an unpublished model from our colleagues at AFD. To complete our economic module, we then introduce an active public sector, once again motivated by unpublished AFD research, and loosely based on the research of Costa Lima et al. (2014) [18]. As in their 2014 paper, we attempt to curb a fall in private profits through a corresponding increase in government expenditures. We wish to show that hopefully, the introduction of government spending can prevent convergence to the “bad” equilibrium.

3 The proposed climate-economy model

The following subsections present a model largely based off the research done by Bovari et al. [2] and [1], with the new additions discussed in Section 2. Note that we do not alter any of the emissions and abatement decisions (Subsection 3.9) presented in Bovari et al. (2018) [1], and we also choose to leave the climate and damages modules untouched as well, following Nordhaus’ DICE model [11] (Subsection 3.10), as well as the research of Dietz and Stern (2015) [14] (Subsection 3.11).

3.1 Accounting framework

In this section, we introduce a simplified version of the economy, in which there are five different sectors: households, firms, banks, a central bank (or network of central banks), and the public sector. Observing Tables 1, 2, and 3, we can investigate the balance sheets, income statements, and transaction flow matrices of the economy. Note that all entries in the country’s balance sheet are measured in nominal monetary amounts, whereas transaction entries and items on the flow of funds sheet are measured in monetary units per unit of time.

We follow the assumption made in Bovari et al. (2017) [2] which states that households cannot take out bank loans, and we further simplify by assuming that households can only hold deposits, denoted by $M^h$. Firms hold deposits, $M^f$, and the stock of capital and its inventories in nominal terms, denoted by $pK$ and $pV$, respectively. The only liabilities of firms are bank loans, $L^c$, which serve as assets for the banking sector in each country. A summarized version of this information can be found in Table 4, the balance sheet for firms in each economy.

As previously mentioned, the banking sector holds firms’ loans as assets, as well as as domestically-distributed government bonds $B^b$. The deposits of households and firms, denoted by $M = M^h + M^f$, are held by banks as liabilities, but a constant proportion of these deposits are then held as reserves (assets), $Res$, by the central bank. Any remaining financing needs of the banks are satisfied by the central bank, through advances $AV$, which are liabilities. We can view the balance sheet for the banking sector in Table 5.

In Table 6, we observe the balance sheet for the country’s central bank. We observe that the central bank holds government bonds, $B^{cb}$, and advances, $AV$, as assets, and reserves, $Res$, as liabilities. The aggregate public sector completes the economy, and since it has no assets, and only distributes bonds to the domestic banks and central bank as liabilities, its extremely simple balance
sheet is not given. We assume that interest is earned on each of the monetary items on the balance sheet, except for reserves.

<table>
<thead>
<tr>
<th>Balance Sheet</th>
<th>Households</th>
<th>Firms</th>
<th>Banks</th>
<th>Public Sector</th>
<th>Central Bank</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Stock</td>
<td>pK</td>
<td></td>
<td></td>
<td>pK</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inventories</td>
<td>pV</td>
<td></td>
<td></td>
<td>pV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deposits</td>
<td>M^h</td>
<td>M^f</td>
<td>−M</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Loans</td>
<td>−L^c</td>
<td>L^c</td>
<td></td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Bonds</td>
<td>B^b</td>
<td>−B</td>
<td>B^cb</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Advances</td>
<td>−AV</td>
<td></td>
<td>AV</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Reserves</td>
<td>Res</td>
<td></td>
<td>−Res</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Sum (net worth)</td>
<td>S^h</td>
<td>X^f</td>
<td>X^b</td>
<td>−B</td>
<td>X^cb</td>
<td>X</td>
</tr>
</tbody>
</table>

Table 1: Balance sheet for country’s economy

<table>
<thead>
<tr>
<th>Transactions</th>
<th>Households</th>
<th>Firms</th>
<th>Banks</th>
<th>Public Sector</th>
<th>Central Bank</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>−pC</td>
<td>pC</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Investment</td>
<td>−pI</td>
<td></td>
<td>−pI</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Govt. Spend.</td>
<td>pG</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>[GDP]</td>
<td>[pY]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[pY]</td>
</tr>
<tr>
<td>Wages</td>
<td>W</td>
<td>−W</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Capital depr.</td>
<td>−δ^K pK</td>
<td>δ^K pK</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Carbon taxes</td>
<td>−pT^b</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Non-carbon taxes</td>
<td>−pS^f</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Abatement subsidies</td>
<td>−pI^f</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>CB profits</td>
<td>Π^cb</td>
<td></td>
<td></td>
<td>Π^cb</td>
<td>−Π^cb</td>
<td>0</td>
</tr>
<tr>
<td>Int. on loans</td>
<td>r^LM^h</td>
<td>−r^L^c</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Int. on deposits</td>
<td>r^M^h</td>
<td></td>
<td></td>
<td>−r^M^h</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Int. on bonds</td>
<td>r^B^b</td>
<td></td>
<td></td>
<td>−r^B</td>
<td>r^B^cb</td>
<td>0</td>
</tr>
<tr>
<td>Firms’ dividends</td>
<td>Π^f</td>
<td></td>
<td></td>
<td></td>
<td>−Π^f</td>
<td>0</td>
</tr>
<tr>
<td>Sum (balance)</td>
<td>S^h</td>
<td>Π^fr</td>
<td>−pI + δ^K pK</td>
<td>S^b</td>
<td>S^g</td>
<td>S^cb</td>
</tr>
</tbody>
</table>

Table 2: Transactions in the given economy

### 3.2 Output and inventories

First assume a Leontieff-type production function for a given country’s economy, and then suppose that a good is produced to a potential level $Y^0$ of output using available labour, $N$, and capital, $K$, such that

$$Y^0 = \min \left\{ \frac{K}{\nu}, aN \right\}. \quad (1)$$

Here, $1/\nu$ and $a$ represent capital productivity and labour productivity, respectively, and we assume that $Y^0 = \frac{K}{\nu} = aL$. In this model, the explicit modelling of consumption is accompanied
### Table 3: Flow of funds in economy

<table>
<thead>
<tr>
<th></th>
<th>Households</th>
<th>Firms</th>
<th>Banks</th>
<th>Public Sector</th>
<th>Central Bank</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in capital stock</td>
<td>$pK$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$pK$</td>
</tr>
<tr>
<td>Change in inventories</td>
<td>$\dot{M}^h$</td>
<td>$\dot{M}^f$</td>
<td>$\dot{M}$</td>
<td></td>
<td></td>
<td>$p\dot{V}$</td>
</tr>
<tr>
<td>Change in deposits</td>
<td>$\dot{M}^b$</td>
<td>$\dot{M}^f$</td>
<td>$\dot{M}$</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Change in loans</td>
<td>$-\dot{L}^e$</td>
<td>$\dot{L}^c$</td>
<td>$\dot{L}^c$</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Change in bills</td>
<td>$\dot{B}^b$</td>
<td>$-\dot{B}$</td>
<td>$\dot{B}^{cb}$</td>
<td>$\dot{B}^{cb}$</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Change in advances</td>
<td>$-\dot{A}V$</td>
<td>$\dot{A}V$</td>
<td>$\dot{A}V$</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Change in reserves</td>
<td>$\dot{\text{Res}}$</td>
<td>$-\dot{\text{Res}}$</td>
<td>$\dot{\text{Res}}$</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Sum (savings)</td>
<td>$S^h$</td>
<td>$\Pi^{fr}$</td>
<td>$S^b$</td>
<td>$S^g$</td>
<td>$S^{cb}$</td>
<td>0</td>
</tr>
<tr>
<td>Change in net worth</td>
<td>$S^h$</td>
<td>$\Pi^{fr}$</td>
<td>$S^b$</td>
<td>$S^g$</td>
<td>$S^{cb}$</td>
<td>$(p\dot{K} + p\dot{K}) + (p\dot{V} + p\dot{V})$</td>
</tr>
</tbody>
</table>

by an assumption that Say’s law does not hold. This causes a disequilibrium in the goods market, in which aggregate domestic production in some country is no longer equal to the domestic demand for goods. In order to explore the economic effects of this disequilibrium, we examine the concept of planned and unplanned changes in inventory, while following the model in Grasselli and Nguyen-Huu (2018) [16]. Given the level of expected sales/output of firms, denoted by $Y^e$ (whose dynamics will be defined shortly), we define the total output of firms by

$$Y^p = (1 - D^Y)(1 - A)(Y^e + I^p). \quad (2)$$

Here, $I^p$ represents planned changes in inventory. We also account for damages to output caused by climate change, $D^Y$, and losses resulting from climate change abatement activities, $A$. As in Bovari et al. (2017) [2], capital evolves according to

$$\dot{K} = I^k - \delta D^K K. \quad (3)$$

Given consumption of domestic goods (defined later), real capital investment, $I^k$, and government spending, $G$, total domestic demand is given by

$$Y^d = C + I^k + G. \quad (4)$$

Then, changes in the level of inventory - which can alternatively be thought of as investment in inventory - can be determined by the difference in total aggregate output and total domestic demand, such that

$$\dot{V} = I^p + I^u := Y^p - \frac{Y^d}{p}, \quad (5)$$

where $I^u$ denotes any unplanned changes in inventory. We can also observe that $I^u$ is thus given as an accommodating variable, accounting for any discrepancies in the level of total output compared to the level of expected output:

$$I^u = \dot{V} - I^p = (Y^p - \frac{Y^d}{p}) - (Y^p - Y^e) = Y^e - \frac{Y^d}{p}. \quad (6)$$
Finally, total investment is given by the sum of real capital investment and any changes in the investment in inventory; that is,

\[ I^T = I^k + I^p + I^u. \]  (7)

### 3.3 Prices, profits, and behavioural rules

The price inflation dynamics are represented by the change in respective consumption price over time:

\[ i = \frac{\dot{p}}{p} := \eta p \left( (1 + \mu) \left( \frac{c}{p} + \nu p \right) - 1 \right). \]  (8)

The multiplication of the markup price, \( \mu \), with the cost of labour to price ratio, \( \frac{c}{p} \), in a country, relaxed by \( \eta p \) also expresses inflation, as evidenced by the rightmost expression in (8). The markup price is given by

\[ \mu = \mu_0 + \mu_1 \left( f^d - \frac{V}{Y^e} \right), \]  (9)

where we assume that a high ratio of inventories to expected output, \( Y^e \), relative to the desired fraction of expected output allocated towards inventories, \( f^d \), will drive prices down. Turning our attention to profits earned by firms, we first observe that the gross and expected profit before dividends paid to households is given by

\[ \Pi^f = pY^d - wL - \delta_{DK} pK + r^M M^f - r^e L^e + p\Upsilon \]  (10)

\[ (\Pi^f)^e = pY^e - wL - \delta_{DK} pK + r^M M^f - r^e L^e + p\Upsilon. \]  (11)

Gross nominal profit is defined as output minus production costs and interest from loans, plus net transfers and interest earned from deposits:

i Money wage bill, given by \( wL \).

ii Capital depreciation, given by \( \delta_{DK} pK \), where \( \delta_{DK} = \delta + D^K \). Note that \( \delta > 0 \) denotes the depreciation rate of capital and \( D^K \) is the fraction of capital destroyed by climate change.

iii Interest earned from deposits, given by \( r^M M^f \), where \( r^M \) represents the short-run nominal interest rate on deposits, and \( M^f \) represents the nominal aggregate amount of deposits.

iv Interest paid on loans, given by \( r^e L^e \), where \( r^e \) denotes the short-run nominal interest rate on loans, and \( L^e \) represents the nominal aggregate amount of firms’ loans.

v Net public transfers, given by \( p\Upsilon = p(S^f - T^f) \); i.e. subsidies from public sector minus taxes.

The profit share, \( \pi^f \), is defined as the ratio of nominal corporate profits to GDP, and is given by

\[ \pi^f = \frac{\Pi^f}{pY^p}. \]  (12)

Then, the fraction of gross and expected profit paid out to households in dividends (provided that \( \Pi^f > 0 \)) is given by the dividend function \( \Delta(\pi^K) \in (0, 1) \). Therefore, we have that
and the retained earnings and expected retained of firms are thus given by

\[
\Pi^{fr} = \Pi^f - \Pi^{fd}
\]
\[
(\Pi^{fr})^e = (\Pi^f)^e - (\Pi^{fd})^e.
\]

(15)

We can define the actual and expected profit rates (also known as the actual and expected return on assets for firms) by the following equations:

\[
\pi^K = \frac{\Pi^f}{pK}
\]
\[
(\pi^K)^e = \frac{(\Pi^f)^e}{pK}.
\]

(17)

(18)

Then, simplifying the long-run growth rate \( g^*(u, (\pi^K)^e) \) presented in Grasselli and Nguyen-Huu (2018) [16] to be equal to the growth rate of labour productivity, we obtain the following dynamics for expected output:

\[
\dot{Y}^e = \alpha^{lg}Y^e + \eta^e (Y^d - Y^e).
\]

(19)

As in [16], firms are assumed to adjust their short-run expectations based on the observed level of demand, and thus the parameter \( \eta^e \) acts as the speed of convergence to the observed level of output demand. Furthermore, by following their assumption (originally introduced in Franke, 1996 [19]) that firms wish to maintain inventories at some level \( V^d = f^dY^e \) (where \( f^d \in [0, 1] \)), we arrive at a similar expression for planned changes in inventory, given by

\[
I^p = \alpha^{lg}V^d + \eta^d(Y^d - V).
\]

(20)

Here, we observe that firms adjust their short-term expectations based on the observed level of inventory, and therefore, \( \eta^d \) represents the speed of convergence to this level of inventory.

### 3.4 Firm financing

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Stock ((K))</td>
<td>Loans ((L^c))</td>
</tr>
<tr>
<td>Inventories ((V))</td>
<td></td>
</tr>
<tr>
<td>Deposits ((M^f))</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Balance sheet for firms

The total financing needs of firms are represented by the difference in their desired investment and their expected profits. These needs are satisfied by domestic loans, and we begin by assuming
that firms face no rationing by domestic banks. With this information, we present the loan dynamics in a given economy as

\[ \dot{L}^c = p\dot{K} + r^cL^c = p(I^k - \delta_D K) + r^cL^c \]  

(21)

where real capital investment is given by

\[ I^k = \kappa((\pi^K)^e)K. \]  

(22)

The investment function, \( \kappa(\cdot) \), depends on the expected profit rate, and follows the specification presented in Bovari et al. (2018) [1]. Then, we assume that deposits evolve according to

\[ \dot{M}^f = (\Pi^f)^e + r^cL^c, \]  

(23)

and so we are able to model the aggregate credit demand of firms as the difference between firms’ loans and deposits. Note that the financing of firms, in terms of the expressions for corporate loans and deposits, follows the work of Grasselli and Lipton (2018) [17]. Then, the modelling of credit demand, which is not rationed, can be equivalently expressed as the total financing needs for firms:

\[ \dot{D} = \dot{L}^c - \dot{M}^f = p\dot{K} - (\Pi^k)^e, \]  

(24)

where the debt-to-output ratio then follows as

\[ d = \frac{D}{pY^p}. \]  

(25)

### 3.5 Bank financing

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government Bonds ((B_{1,1}^B))</td>
<td>Deposits ((M))</td>
</tr>
<tr>
<td>Loans to firms ((L^c))</td>
<td>Advances from CB ((AV))</td>
</tr>
<tr>
<td>Reserves at CB ((Res))</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Balance sheet for banks

Keeping in line with Grasselli and Lipton (2018), as well as unpublished work from our colleagues at AFD, we assume that there is a constant required reserves ratio for the banking sector, denoted by \( f^M \in [0, 1] \), such that

\[ Res = f^MM \quad \text{and} \quad \dot{Res} = f^M\dot{M}. \]  

(26)

Then, suppose that banks purchase a constant fraction of the bonds supplied by the public sector. Denoting this fraction by \( \Omega \), we thus observe that

\[ \dot{B}^b = \Omega \dot{B}. \]  

(27)

From the balance sheet, we can then write the profits of banks as

\[ \Pi^b = r^cL^c + r^B B - r^M M - r^A AV. \]  

(28)
The central bank is thus a lender of last resort, and through the central bank’s admission of advances, we achieve stock-flow consistency. We model this residual below, and note that it is equal to the total remaining financing needs of banks:

\[ AV = TFN^b = Res = (1 - \Omega)B^b. \] (29)

### 3.6 Public sector

We assume that government spending (excluding abatement subsidies) will be characterized by a proportion of private domestic demand:

\[ G = \psi(C + pI^k). \] (30)

We follow the specification of AFD in supposing that the proportion of private demand allocated for government expenditures depends on the current employment rate, public deficit ratio, and public expenditure level, all relative to their desired levels in equilibrium, thus evolving according to

\[ \dot{\psi} = \psi \left( \beta^G_1 (\lambda^{eq} - \lambda) - \beta^G_1 \left( \frac{B}{Ye} - G_{def}^{eq} \right) - \beta^G_2 (\psi - \psi^{eq}) \right), \] (31)

where \( \beta^G_1, \beta^G_2 \) are scaling parameters. Then, excluding carbon taxes levied on firms, we suppose that the government taxes wages and bank profits. Tax revenues are thus given by

\[ T = \tau_w wL + \tau_b \Pi_b. \] (32)

Finally, considering carbon taxes, abatement subsidies, and central bank profits, we obtain the expression for the government deficit, which is financed through the supply of government bonds:

\[ \dot{B} = (G + S^f) - (T + T^f) + rB B - \Pi_{cb}. \] (33)

The government deficit-to-output ratio is thus given by

\[ G_{def} = \frac{\dot{B}}{Ye}. \] (34)

### 3.7 Central bank

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government bonds (( B^{cb} ))</td>
<td>Reserves (( Res ))</td>
</tr>
<tr>
<td>Advances to banks (( AV ))</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Balance sheet for central bank

Assuming that the central bank sets the interest rate on bonds and advances by using a variant of the Taylor Rule [20], we observe that

\[ r^B = i + \theta^B \left( \frac{B}{pYe} \right) = r^A. \] (35)
We make the simplifying assumption that \( r^B = r^A \), and allow for these interest rates to take both inflation and the expected public deficit-to-output ratio into account. Then, we allow for a small negative perturbation to this nominal interest rate, thus determining the short-run nominal interest rate on deposits:

\[
    r^M = r^A - \iota^M. \tag{36}
\]

From here, we assume that the average fixed cost in the banking sector is simply the average rate that equates the interest earned on advances and deposits, and the nominal monetary amount of advances and deposits themselves. We define the average fixed cost, \( AFC \), as

\[
    AFC = \frac{r^A AV + r^M M}{AV + M}, \tag{37}
\]

and then allow for a Taylor-inspired short-term lending rate, expressed by

\[
    r^c = \max\{i, AFC + prem\} \tag{38}
\]

where \( prem \) represents a lending rate premium, designed to encourage borrowing over saving. Returning to the bond market, we assume that the central bank clears the market by absorbing the supply of government bonds in excess of demand:

\[
    \dot{B}^b = \dot{B} - \dot{B}^b. \tag{39}
\]

3.8 Labour markets and households

A country’s workforce, \( N \) is assumed to grow logistically according to

\[
    \dot{N} = \delta_N N \left( 1 - \frac{N}{N} \right), \tag{40}
\]

where \( N \) represents the upper limit of the workforce, and \( \delta_N \) is the growth rate of a country’s workforce dynamics. Similarly, population, \( NG \), is given by

\[
    \dot{NG} = \delta_{NG} NG \left( 1 - \frac{NG}{NG} \right). \tag{41}
\]

Labour is assumed to be the abundant factor of production. As a result, we assume that labour is hired at full capacity, such that

\[
    L = \frac{Y^p}{a(1 - D^V)(1 - A)}, \tag{42}
\]

and so the employment rate is endogenously given by \( \lambda = \frac{L}{N} \). For workers, the growth in labour productivity and nominal wages is modeled by

\[
    \dot{a} = \alpha \cdot a \tag{43}
\]

\[
    \dot{w} = w(\varphi(\lambda) + m\bar{m}), \tag{44}
\]

where \( \varphi(\lambda) \) is a function depicting a short-run Phillips-curve as in Goodwin’s original paper (Goodwin 1967) [6], with linear form \( \varphi(\lambda) = \phi_0 \lambda + \phi_1 \). Furthermore, we follow Grasselli and Nguyen

10
Huu (2015) [21] and include inflation in the wage bargaining dynamics. Note that the real wage, equivalent to the unit labour cost divided by price is defined as

$$\omega = \frac{wL}{pY^p} = \frac{c}{p}.$$  \hfill (45)

Households receive income through labour, they earn interest on their deposits, and receive a portion of firms’ profits through dividend payments. They do not hold financial assets such as bonds or loans, and spend solely on consumption goods. Total household savings are thus given by

$$S^h = (1 - \tau^w)wL + r^M M^h + \Pi^d - C,$$  \hfill (46)

where household consumption is expressed as

$$C = m^1((1 - \tau^w)wL + r^M M^h + (\Pi^d)^c) + m^2 M^h.$$  \hfill (47)

Note that the first term in the consumption expression represents all consumption of income, where $m^1$ represents the proportion of income consumed. The second term represents the idea of “emergency consumption” in which households consume a small portion, $m^2$, of their savings. Lastly, since households can only hold deposits, we can say that household savings are equivalent to the change in deposits:

$$\dot{D}^h = S^h.$$  \hfill (48)

### 3.9 Emissions, carbon taxes, and abatement

Turning our attention to emissions, we know that given a carbon pricing function, $p_C$, firms will determine a desired emission reduction rate $n \in (0, 1)$. Industrial emissions in each country are modelled by

$$E_{ind} = \frac{\sigma(1 - n)Y^p}{(1 - D^i)(1 - A)}$$  \hfill (49)

where $\sigma_i > 0$ is the carbon intensity of the economy, with initial growth rate function $g_{\sigma_i}$. These functions are coupled and are given by

$$\dot{\sigma} = g_{\sigma_i} \sigma_i,$$  \hfill (50)

$$g_{\sigma} = \delta_{g_{\sigma}} g_{\sigma},$$  \hfill (51)

where $\delta_{g_{\sigma}} < 0$ is a parameter denoting the variation rate of the growth of emission intensity. Then, as in Nordhaus (2016) [12], the abatement cost function in a given economy, $A$, is defined as a function of the emissions reduction rate, $n$. It is then normalized by the emission intensity of the economy, $\sigma$, and the price of a backstop technology, $p_{BS}$:

$$A = \frac{\sigma p_{BS}}{1000 \times \theta n^{\theta}}.$$  \hfill (52)

The parameter $\theta > 0$ controls the convexity of the cost. The price of the backstop technology then grows according to

$$\dot{p}_{BS} = \delta_{p_{BS}} p_{BS}.$$  \hfill (53)
Examining the public sector and its role in abatement activities, we introduce carbon taxes paid to the two governments by firms, and abatement subsidies paid back to the firms from the public sector. Following the work of Bovari et al. (2018) [1], a carbon tax will be levied on the emissions of firms, such that

$$T_f = p_C E_{ind}.$$ (54)

and a fraction, $s_A$, of abatement costs paid by firms may be subsidised by the government, resulting in a national transfer of

$$S_f = s_A A Y^0.$$ (55)

Therefore, net transfers from the public sector to the private sector are denoted by

$$\Upsilon = S_f - T_f.$$ (56)

The research of Bovari et al. (2018) [1] assumes two different carbon pricing strategies, both of which we follow. The first is a low carbon tax with growth rate 2% per year (denoted $p_{C_{low}}$, in line with the Baseline scenario of Nordhaus (2014) [22]. The second is a high carbon tax (denoted $p_{C_{high}}$), following the recommendation of the Stern-Stiglitz report (Stern and Stiglitz, 2017 [3]). We follow the upper barrier of the corridors presented in their report: from US$40-80/tCO_2$ by 2020 to US$80-100/tCO_2$ by 2030, with linear interpolations of the growth rate of $p_C$ until 2100.

Firms then choose their abatement emissions rate depending on the carbon price, $p_C$, the cost of backstop technology, $p_{BS}$, and the public subsidization of green technology, $s_A$ according to

$$n = \min \left\{ \left( \frac{p_C}{(1-s_A)p_{BS}} \right)^{\frac{1}{\theta-1}}, 1 \right\}$$ (57)

### 3.10 Climate Module

The climate module presented below follows the same framework laid out by Nordhaus in his DICE model [11], and later reintroduced by Bovari et al. in [2] and [1]. Global CO$_2$ emissions are assumed to be the sum of industrial and land-use emissions, where land-use emissions are assumed to be exogenous and decrease at the rate $\delta_{E_{land}}$.

$$E = E_{ind} + E_{land}$$ (58)

$$\dot{E}_{land} = \delta_{E_{land}} E_{land}$$ (59)

The carbon cycle is then modeled in three layers. CO$_2$ emissions can accumulate in the atmosphere (CO$_2^{AT}$), the upper ocean and biosphere (CO$_2^{UP}$), or the lower ocean (CO$_2^{LO}$). The dynamics of CO$_2$ accumulation progress as such:

$$
\begin{pmatrix}
\text{CO}_2^{AT} \\
\text{CO}_2^{UP} \\
\text{CO}_2^{LO}
\end{pmatrix} =
\begin{pmatrix} E_T & -\phi_{12} & \phi_{12} C_{UP}^{AT} \\
-\phi_{12} & \phi_{12} C_{UP}^{AT} - \phi_{23} & 0 \\
\phi_{23} & -\phi_{23} C_{LO}^{UP} & -\phi_{23} C_{LO}^{UP}
\end{pmatrix}
\begin{pmatrix}
\text{CO}_2^{AT} \\
\text{CO}_2^{UP} \\
\text{CO}_2^{LO}
\end{pmatrix}
$$ (60)

where $C_{ij}^j = \frac{C_{preind}^j}{C_{preind}^{AT}}$ for $i,j \in \{AT, UP, LP\}$ and $\phi_{12}, \phi_{23}$ are parameters, as in Bovari et al (2018) [1]. The accumulation of CO$_2$ then increases radiative forcing, $F$, of CO$_2$. This is modeled
as follows, where $F_{dbl}$ is an exogenous parameter that represents the effect on forcing of a doubling of pre-industrial CO$_2$ levels, and $F_{exo}$ increases at an exogenous rate over time.

$$F_{ind} = \frac{F_{dbl}}{\log(2)} \log \left( \frac{CO_2^{AT}}{C_{AT_{preind}}} \right)$$

(61)

$$F = F_{ind} + F_{exo}$$

(62)

Then, we model the effects of radiative forcing on temperature, given by $T$. Mean temperature is divided into two layers, the atmosphere, land, and upper ocean layer, $T$, and the lower ocean layer, $T_0$. The heat capacities of each layer are given by $C$ and $C_0$, while $\gamma^*$ represents the heat transfer between layers. The equilibrium climate sensitivity is given by $S$, and thus temperature in each layer evolves according to

$$\dot{T} = \frac{F - F_{dbl} S T - \gamma^*(T - T_0)}{C}$$

(63)

$$\dot{T}_0 = \frac{\gamma^*(T - T_0)}{C_0} .$$

(64)

### 3.11 Damages

Following the research of Dietz and Stern (2015) [14], we assume that rising temperatures can reduce both output and capital, thereby further reducing output. Temperature factors into the damage function originally presented in [14], which is given by

$$D = 1 - \frac{1}{1 + \pi_1 T + \pi_2 T^2 + \pi_3 T^\zeta}$$

(65)

The total damages are then allocated between output and capital according to a proportion $f_k \in [0,1]$: 

$$D^K = f_k D$$

(66)

$$D^Y = (1 - f_k) D$$

(67)

In previous research, three different specifications of the damage curve are employed. As illustrated in Bovari et al. (2017) [2], the Nordhaus specification, from [11], is the least convex, while The Weitzman and Stern specifications are progressively more convex [13] [14]. Our simulations observe damages incurred under the Nordhaus and Stern specifications, assuming a discrete array of the fraction of damages allocated to capital, of $f_k = \{0, 1/3\}$.

### 4 Results

Due to the high-dimensionality of our system, performing any sort of analytical work is near impossible. In order to best understand the intricacies of this model, we must observe various numerical simulations of different damages and policy scenarios. While the ultimate goal of the model is to analyze the effects of rising temperatures on the global economy, we present a simplified version of the model, in which we only observe the future effects that climate change will have on the United States. The initial year for all simulations is 2016, and the set of initial conditions and parameters that were used to perform our simulations can be found in the Appendix 6.
4.1 Economic module

While the many components of the economic module are derived from previous published work, the amalgamation of these components allows for a novel economic module. A good starting point is thus a quick analysis of the results of this module. In this subsection, we assume that firms produce and emit as they would under normal circumstances, and that temperature does indeed rise. We simply set damages to zero, and thus allow for the economy to grow undeterred. A low carbon tax with a growth rate of 2% per year is used.

![Graphs of economic module ratios](image)

Figure 1: Plots of important ratios from 2016 to 2100, including the wage share ($\omega$), the employment rate ($\lambda$), the profit share ($\pi^f$), and the private debt ratio ($d$). Simulations employed no damage curve, and a low real carbon tax.

At first glance, Figure 1 already begins to show troubling signs. While the wage share seems to stabilize at a reasonable ratio of approximately 0.555, the employment rate oscillates until shortly after 2040, before entering a steady decline (albeit to an economically sensible 94%). The corporate debt-to-output ratio increases monotonically, while the profit share decreases monotonically; both are worrying signs for the private sector. Other than the wage share, none of the variables presented appear to be converging to an equilibrium level. The finite, non-zero wage share and employment rate are contradicted by a monotonically increasing private debt ratio, thus negating the existence of the traditional “good” equilibrium (at least in this parameter space).

Figure 2 confirms our earlier worries; we seem to be approaching the “explosive” equilibrium discussed in Bovari et al. (2018) [1]. The wage share is the variable that is least changed over time. What originally appeared to be convergence to a steady-state, however, is unfortunately untrue, as the wage share begins a steady decline after the year 2200. The (now extremely) brief oscillations is
employment rate were also misleading, as the employment level decreases rapidly over the coming centuries. Meanwhile, the private sector experiences a profit share that approaches zero, and a private debt ratio that well exceeds the dangerous \( d = 2.7 \) threshold.

Thankfully, there are some sensible explanations for why our economy is seemingly destined for a massive recession at the turn of the next century. These answers mainly revolve around a deficient pricing mechanism, which sees the commodity price in our model barely increase in the coming centuries (see Figure 3). This corresponds with extremely low (but positive) levels of inflation in the next hundred years, followed by a steady deflationary pattern afterwards, culminating in negative inflation by 2400.

The lending rate - as well as the other interest rates - follow the same pattern as the inflation rate, while real output exhibits a balanced growth path. The growth rate of real output is suppressed, and therefore does not exhibit the properties of balanced growth paths outlined in Solow (1956) [23]. We suspect that the suppression of the real growth rate is caused by stunted prices, thus creating a negative feedback loop within our model, pushing our economy into the observed recession. This is discussed more in Section 5.

4.2 Monte Carlo simulations

Uncertainties regarding climate growth, damages, and abatement activities are addressed in this section. There is an extensive array of literature (see Knutti et al. 2017 [24], for example) that
Figure 3: Plots from 2016 to 2400, of prices ($p$), the inflation rate ($i$), the nominal, short-term interest rate ($r^c$), and the real output, in trillions of 2015 USD ($Y^p$). Simulations employed no damage curve, and a low real carbon tax.

discusses three of the main drivers of the climate-economy system presented in Bovari et al. (2018) [1]. Specifically, these parameters are the long-term growth rate of labour productivity, $\alpha^{lg}$, the equilibrium climate sensitivity, $S$, and the inertia of the climate system, which is dependent on the size of the intermediate carbon reservoir, $C_{UP_{preind}}$.

We follow the same probability density function (hereafter pdf) specifications as those outlined in Bovari et al. (2018) [1] (which follow the work of Nordhaus, 2016 [12]). Alternatively, we depart from their work by choosing to not examine the sensitivity of $\alpha^{lg}$. There is little to no research regarding regional uncertainties of these climate-economy parameters, and since we are examining a USA-centric version of the model, we thus cannot utilize the pdf for $\alpha^{lg}$.

4.2.1 Equilibrium climate sensitivity and the carbon cycle

Bovari et al. (2018) [1] assumes that the equilibrium climate sensitivity, $S$, follows a log-Gaussian distribution with mean $\mu = 1.107$ and standard deviation $\sigma = 0.264$; i.e. $S \sim \log -N(1.107, 0.264)$. The choice for this pdf is motivated by the Bayesian estimates of Gillingham et al. (2015) [25].

Turning our attention to the climate cycle, we assume the same specification as Bovari et al. (2018) [1] for the pdf of the intermediate reservoir size, denoted by $C_{UP_{preind}}$. This parameter is arguably the most important element of the carbon cycle, as it ultimately determines the inertia of the climate cycle. Following the work of Nordhaus (2016) [12], Bovari et al. postulate that $C_{UP_{preind}}$ follows a log-Gaussian distribution with mean $\mu = 5.8855763$, and standard deviation
\[ \sigma = 0.2512867; \text{ i.e. } C_{UP_{preind}} \sim \log \mathcal{N}(5.8855763, 0.2512867). \]

### 4.2.2 Climate risk

We test two damage function specifications, the milder damage function provided by Nordhaus (2016) [12], and the more convex specification introduced by Dietz and Stern (2015) [14]. Since climate change may cause damages to output directly or indirectly (through damages to capital stock, in our case), we also vary the proportion of damages allocated to capital. As in Bovari et al. (2018) [1], we consider the estimate of Dietz and Stern (2015) [14] (based on the results of Nordhaus and Boyer, 2000 [26]), in which they assume that a third of damages are allocated to capital. For the sake of comparison to the Baseline scenario of Nordhaus (2014) [22], we also consider that no damages are directly allocated to capital. To summarize, we choose to test \( f_k = 0 \) for both the Nordhaus damage function, and \( f_k = 1/3 \) for the Stern damage function.

### 4.2.3 Policy scenarios

The three scenarios outlined in Bovari et al. (2018) [1] will be considered for a full Monte Carlo approach. We briefly summarize the three scenarios below:

1. **No policy.** This scenario is based on a Monte Carlo simulation with the Nordhaus damage function. Public intervention is minimized; a weak carbon tax with a growth rate of 2% per year is implemented. This scenario serves as a control, and is motivated by the Baseline scenario of Nordhaus (2014) [22].

2. **Carbon tax.** This scenario is based on a Monte Carlo simulation with the Stern damage function. The carbon tax follows the recommended path outlined by the Stern-Stiglitz commission (Stern and Stiglitz, 2017) [3], previously denoted by \( p_{C_{\text{high}}} \).

3. **Carbon tax and subsidy.** This scenario is based on the Carbon tax scenario, with an additional 25% public subsidy on abatement technology costs (i.e. \( s_A = 0.25 \)).

### 4.2.4 Monte Carlo simulation results

All three scenarios are simulated through 200 Monte Carlo runs, from 2016 to 2100. In Figures ?? and 5, the shaded blue region represents the [0.025;0.975] probability interval in the given scenarios, with the solid blue line representing the median value for each variable.

The **No policy** scenario yields mostly uninteresting results, that are very similar to the original economic-only module. From Figure 4, we can observe that the wage share appears to stabilize at a sensible value of 0.555 in the short-term, before decreasing in the long-run (well after 2100), all with very little deviation. There is also almost no deviation in the private debt ratio and the profit share, as they exhibit the same monotonic dynamics to that of the original damage-free scenario considered earlier.

There is little deviation in the employment rate in **No policy** scenario, and it in fact has a slightly greater median level than the employment rate in the damage-free economic module. Output grows steadily, as before, while emissions drop to below 1 GtCO\(_2\) by the end of 2100. The most important variable that experiences significant deviation is the temperature anomaly, which ranges from 1.5 °C to over 3 °C.

As was the case in the aforementioned economic, damage-free module, a dangerously low inflation rate, caused by stagnant prices, is most likely the major cause of (eventual) economic downturn in the model. The abnormally low inflation rate consequently leads to suppressed interest rates,
and while wages do increase at a desirable speed (until 2100), the rest of the model suffers as a result.

The Carbon tax and Carbon tax and subsidy scenarios produce very similar results, and so we only show the simulated results for the latter scenario (Figure 5). Both policy scenarios are accompanied by a more aggressive damage function, with one-third of the climate-induced damages entering the system through the capital channel. As a result, the first observation we can make is that the dynamics exhibited by the variables in the No policy scenario and the economic-only module are simply exacerbated in the more intensive policy scenarios.

The median wage share has increased, and does so with greater deviation, especially in the second half of the century. These increasing wages, along with stagnant prices (and thus abnormally low inflation, once again), push the median profit share down to $\pi^f = 0.1$, with the worst-case scenario of $\pi^f = 0$ appearing as a realistic possibility as early as 2085 or so. The unemployment rate rises to a dangerous level of above 15% in the waning years of the century, with an even more drastic decrease in the long run. Output, however, grows as steadily as before (albeit with greater deviation from the mean), and emissions decrease to 0. Temperature evolves similarly to the No policy model, as well as the public deficit-to-output ratio.

The only variable that exhibits vastly different dynamics than before is $d$, the private debt ratio. At first glance, one might worry that our model is inherently wrong, and that a vastly negative debt ratio (or in other words, a private surplus) is a result of a dynamical error. While this should not happen, and while this must be addressed in future research, this makes perfect
Figure 5: [0.025;0.975] probability interval of the Carbon tax and subsidy scenario. The Stern damage specification is used, with $f_k = 1/3$. There were 200 Monte Carlo simulations performed, and the medians are the solid blue lines. Temperature anomaly is given in °C.

In this section, we provide an interpretation of the results, while also discussing future avenues and objectives for research.

### 5.1 Pricing mechanism

As we have observed in all scenarios, many key variables are adversely affected by stagnant prices, to the point that in the long run, our model predicts a major economic recession. The central cause for the stagnant prices (and consequently an inflation rate that is dangerously low, and even negative in the long run) is easy to identify; unfortunately, a possible solution to this problem is less evident.

Our parameter space is such that the only way to drastically increase prices is to raise either increase the price markup, $\mu$, or to increase the scaling parameter $\iota_p$, both in equation 8.
In both cases, these changes decrease the convexity of the pricing path, thus causing inflation to asymptotically approach zero. This means that while prices increase to a greater value, they do so by rapidly increasing in the near-term, before gradually reaching a maximum at the end of the century.

Zero (or near-zero) inflation is the result in this case, and we are thus faced with skyrocketing private debt and a declining wage share. This is accompanied by an increase in the profit share, but also an economically impossible employment rate greater than 1. If we attempt to address both the unreasonably high employment rate and declining wage share by raising wages in the model, we will drive the cost labour up, thus driving prices back down.

All of this points to the need for a better, more exponential pricing mechanism, one that can sustain a positive, greater inflation rate. Higher prices will lead to greater profits in the private sector, which will then be accompanied by a greater profit rate. A greater profit rate will increase firms’ desire to invest, thus increasing their need to borrow. This will prevent the level of corporate debt from exhibiting a drastic, unprecedented decrease, as illustrated in Figure 5. The increased level of investment will lead to greater domestic demand, thus increasing government spending, as well as the profits and output of the private sector.

5.2 Inventories, investment, and consumption

Our simulations predict rising inventory levels towards the end of the century (see Figure 6), a result of an increasing disparity between levels of supply and demand. Domestic production, $Y_p$, grows too quickly relative to domestic demand, $Y_d$, and so in order to reduce that disparity, we must either increase demand or reduce supply. Since one of our next issues will be addressing a rather slow output growth path, our solution must ultimately be centered around increasing demand.

As in Subsection 5.1, a more convex, exponential pricing mechanism may be able to stimulate investment, while also increasing the amount of public expenditures. This would indeed raise demand, but will also come at the cost of raising production (and thus supply), as well. We could raise the amount of government spending, but that may eventually become an unreasonable solution, as our current proportion of domestic demand allocated towards public spending is approximately 30%, and thus in line with recent trends in the United States.

Perhaps a more reasonable solution is to redefine our investment function, taking corporate debt into account. Debt-financed investment is modelled in Bovari et al. (2018) [1], and by taking the level of corporate debt into account, we would not only create a more realistic investment strategy, but also one that increases domestic demand. This could possibly reduce the disequilibrium in the goods market, which may then reduce markup, thus spurring an increase in the convexity of the pricing mechanism. Furthermore, factoring debt cycles into investment (and thus demand) may create the boom-bust market oscillations (to/around some equilibrium) that have been exhibited in much of the Keen-based literature.

5.3 Damages and output

A key observation of our model seems to be that damages do not adversely affect the level of output. One main reason for this is that output simply does not grow at a rate necessary for a vast increase in emissions, and thus a greater increase in temperature. Even when there is very little public intervention, emissions do not grow quick enough to bring about the level of damages that would severely impact the economic module, and once stronger public intervention is introduced, emissions monotonically decrease towards zero.
Figure 6: Plots of the level of inventories ($V$), consumption ($C$), capital investment ($I^k$), and government expenditures ($G$) from 2016 to 2100. Simulations employed no damage curve, and a low real carbon tax.

This unfortunate aspect of the model should be solved once the two previous issues are addressed. Increasing both pricing and demand should increase output, and while that may lead to grave consequences in the climate module (and thus back into the economic module, through the damage functions), it will at least signify that our model is behaving realistically (take a win where you can get one).

6 Conclusion

Our model aims to create a more robust, realistic framework for the economy, and to thus further understand the drivers of economic collapse in the case of climate change. We have built upon the work of Bovari et al. (2017) [2] and (2018) [1] by adding inventory dynamics, explicit consumption, and active public and banking sectors to the economic module. Unfortunately, our model is unable to replicate the rich dynamics of Keen-based, stock-flow consistent literature. Stagnant prices hinder growth, and lead to declining profit shares in the private sector. A low inflation rate ultimately goes to zero in the long run, as does the wage share and employment rate. Even the private debt ratio exhibits negative dynamics, in response to the increasing wages and stagnant prices.

Perhaps the most troubling aspect of our model is its inability to illustrate a growth path that is capable of increasing emissions to the point of a temperature-induced, climate-economic breakdown. We have outlined some possible solutions for this issue, including the introduction of a
pricing mechanism that is more convex, and the introduction of debt-financed investment. A more exponential pricing function will lead to stable inflation and sustained growth, while debt-financed investment should increase demand, reducing the discrepancy in the goods market, and hopefully negating a situation in which the economy enters economic depression sans any climate feedback.

The ultimate goal of this model is to eventually extend it to the desired global scale, and introduce a scenario in which the Global North and Global South are separate, but remain financially linked. This will allow us to disaggregate the wealthier, developed nations, who will be better equipped to deal with potential climate-economic catastrophe, from the poorer, less developed countries, who will bear the immediate brunt of these damages. Understanding the financial linkages between these two regions (through trade, foreign exchange, and foreign direct investment, for example), will enable us to better identify policies that may be able to mitigate the long-term damages of climate change, and create a sustainable future for all.
## A Simulation values for the model

### A.1 Initial conditions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Variable description</th>
<th>Initial condition</th>
<th>Sources and remarks</th>
</tr>
</thead>
<tbody>
<tr>
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## A.2 Calibration parameters

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<th>Symbol</th>
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<th>Sources and remarks</th>
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References


